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WIND-DRIVEN SEA LEVEL CHANGE OF A SHALLOW SEA OVER A CONTINENTAL SHELF

Koji Hidaka

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Wind-Driven Sea Level Change of a Shallow Sea Over a Continental Shelf[1]

by

Koji Hidaka

ABSTRACT

A theory of wind-driven surface slope and level change in a shallow sea close to the coast is given taking into account the earth's rotation and both vertical and horizontal mixing. A wind zone of finite width extending from the coast is assumed and the surface slopes in a steady state are computed at several distances from the coast. If these are pieced together, we can give the surface water level change as a function of the distance from the coast. This research represents a portion of a voluminous work which the author is carrying out concerning the three-dimensional steady motion of water and the surface-contours as generated by a steady wind.

I. Introduction.

The concept of horizontal mixing introduced by C.-G. Rossby (1936) and subsequently developed by R. B. Montgomery and H. U. Sverdrup has presented several important changes and advantages in the physical explanation of various meteorological and oceanographical phenomena which had hitherto been very hard to explain. Montgomery mentioned various evidences which showed that some oceanographical phenomena cannot be explained without taking this concept into account. We can mention the successful

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explanation of the westward intensification of the Gulf Stream and the Kuroshio by this idea demonstrated several years ago by Henry Stommel (1948), Walter H. Munk (1950) and Koji Hidaka (1951). A theory of upwelling recently worked out by Hidaka (1953) is also based on this consideration. The present discussion also consists of an application of this concept and treats the surface form of the sea off a straight coast developed by the effect of steady winds blowing in a certain direction in a finite band within a certain distance from the coast.

The theory of piling-up of water on the coast by the action of the wind was first treated by V. W. Ekman. His explanation consisted of the fact that very close to the coast the steady flow of water driven by wind toward or away from the coast just balances the flow due to the slope current produced by the piling-up on or taking-away of water from the coast. This seems to have been successful in predicting the slope and of the water surface approximately. But since his theory assumes that the velocity and surface slope are uniform in horizontal directions the difficulty is that of how far from the coast the predicted slope is. Present research shows that the slope and level change of the water surface occurs mostly below the wind zone only. Further, Ekman's theory is unable to say how the height of the sea surface varies as we are removed away from the coast. This is mathematically impossible because only the vertical momentum transfer is taken into account and the velocity components and slope of water surface are functions of vertical coordinates z alone. In order to discuss the horizontal variation of these quantities, however, it is necessary to consider horizontal mixing.

The following theory is nothing but a modification of Ekman's theory of wind-driven currents made by introducing the effect of horizontal eddy

viscosity. Still the result has some advantages over the classical theory in explaining various features encountered in the actual sea, especially in enabling us to know the horizontal variation of the velocity components and surface slope. If the complete numerical computation could be worked out, this problem would give a complete structure of water motion produced by the stress of the winds in both deep and shallow seas. However, this would require a great amount of tedious calculation so the complete discussion is left for the future and only the distribution of the surface slope and the change of level in an offshore direction will be treated in this paper. It gives the steady surface slope developed by wind in a sea of finite depth and will be especially applicable to the problem of wind-produced piling-up or lowering of water in continental shelves such as found in the Gulf of Mexico or the North Siberian Shelf.

II. Theory

Consider a straight coast coincident with the axis of y, with the x-axis perpendicular to it in the offshore direction. (Figure 1) Suppose a wind of constant force and direction is bl wing steadily in a belt of limited width L at a certain angle with the coast. Take the z-axis vertically downward.

If a constant wind blows for a sufficiently long time, a steady state will be attained in which the motion of water is independent of time. We assume that the wind stress cannot vary in the y-direction, but may be a function of x. This means that the wind can vary in an offshore direction only. In such a steady state all the vertical and horizontal components of the currents can be determined as functions of x and z only. Of course, the

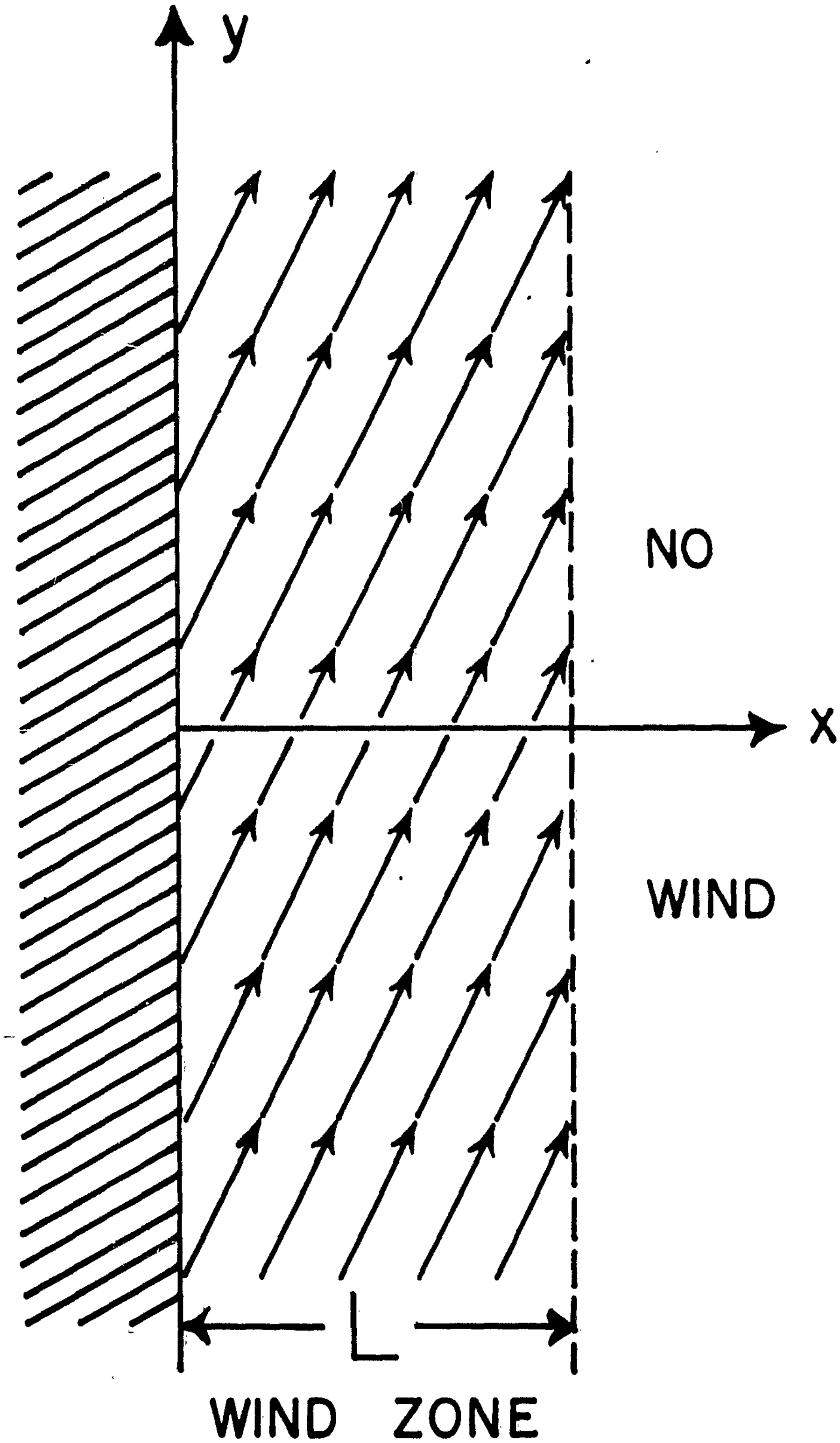

FIG. 1 COAST, WIND ZONE AND CALM AREA.

surface of the sea will not be a plane, but have a slope in the offshore direction, the amount of slope varying as a function of the distance x from the coast. In such a case the hydrodynamical equations of motion are, after several reasonable simplifications,

$$\left.\begin{aligned} &\frac{A_h}{\rho}\frac{\partial^2 u}{\partial x^2} + \frac{A_v}{\rho}\frac{\partial^2 u}{\partial z^2} + 2\omega \sin\varphi\, v - g\frac{d\zeta}{dx} = 0, \\ &\frac{A_h}{\rho}\frac{\partial^2 v}{\partial x^2} + \frac{A_v}{\rho}\frac{\partial^2 v}{\partial z^2} - 2\omega \sin\varphi\, u \qquad = 0 \end{aligned}\right\} \tag{1}$$

where u and v are the horizontal components of the current velocity in the x- and y- directions, ζ the surface elevation depending on x only, the density, A_v and A_h the coefficients of vertical and horizontal mixing (eddy viscosity) of sea water, ω the angular velocity of the earth and φ the geographic latitude. In addition to these, we have the equation of continuity in the form

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

since $\frac{\partial v}{\partial y} = 0$. Here w is the vertical component of currents.

Let the components of wind stress be given by τ_x and τ_y . Because the zone of wind is of finite width within a distance L from the coast, the conditions to be satisfied on the surface of the sea are therefore

$$\left.\begin{aligned} -A_v\frac{\partial u}{\partial z}\Big|_{z=0} &= \tau_x \quad \text{for} \quad 0 \leqq x \leqq L \\ &= 0, \quad \text{for} \quad L < x < \infty; \\ -A_v\frac{\partial v}{\partial z}\Big|_{z=0} &= \tau_y \quad \text{for} \quad 0 \leqq x \leqq L \\ &= 0 \quad \text{for} \quad L < x < \infty \end{aligned}\right\} \tag{3}$$

where τ_x and τ_y may be either constants or functions of x only. On the bottom $z = h$ we must have

$$z = h: u = v = 0 \tag{4}$$

because of the vertical friction. Along the coast which for the sake of simplicity may be supposed to be vertical

$$x = 0: u = v = 0, \tag{5}$$

because of horizontal friction. In the region very far from either coast or the wind region, we have

$$x = \infty: u = v = 0 \tag{6}$$

Let us define D_v and D_h by

$$D_v = \pi\sqrt{A_v/\rho\omega\sin\varphi}\ ;\ D_h = \pi\sqrt{A_h/\rho\omega\sin\varphi}\ . \tag{7}$$

D_v is the "depth of frictional influence" defined by Ekman (1905) in his theory of wind-driven ocean currents, and D_h is a quantity having a dimension of length and may be called "frictional distance". This is a measure of the horizontal turbulence.

If we put

$$\xi = x/D_h \tag{8}$$

the equations in (1) now become

$$\left.\begin{aligned}\frac{\partial^2 u}{\partial \xi^2} + D_v^2\frac{\partial^2 u}{\partial z^2} + \pi^2 v - \frac{\pi^2 g}{\omega \sin\varphi}\frac{d\zeta}{dx} &= 0,\\ \frac{\partial^2 v}{\partial \xi^2} + D_v^2\frac{\partial^2 v}{\partial z^2} - \pi^2 u &= 0.\end{aligned}\right\} \tag{9}$$

In order to solve these equations (9), suppose with Takegami (1934)

$$\left.\begin{aligned}u &= \frac{2}{\pi}\int_0^\infty u_1(z,\lambda)\sin\lambda\xi\, d\lambda,\\ u_1(z,\lambda) &= \int_0^\infty u(\alpha,\lambda,z)\sin\lambda\alpha\, d\alpha\end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned}v &= \frac{2}{\pi}\int_0^\infty v_1(z,\lambda)\sin\lambda\xi\, d\lambda,\\ v_1(z,\lambda) &= \int_0^\infty v(\alpha,\lambda,z)\sin\lambda\alpha\, d\alpha\end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned}\frac{d\zeta}{dx} &= \frac{2}{\pi}\int_0^\infty \gamma(\lambda)\sin\lambda\xi\, d\lambda,\\ \gamma(\lambda) &= \int_0^\infty \frac{d\zeta}{dx}\sin\lambda\alpha\, d\alpha\end{aligned}\right\} \tag{12}$$

Next suppose for the wind stress

$$-A_v\frac{\partial u}{\partial z}\Big|_{z=0} - A_v\frac{\partial v}{\partial z}\Big|_{z=0} = \frac{2}{\pi}\int_0^\infty T(\lambda)\sin\lambda\xi\, d\lambda \tag{13}$$

and

$$\begin{aligned}T(\lambda) &= \int_0^\infty \left\{-A_v\frac{\partial u}{\partial z}\Big|_{z=0} - A_v\frac{\partial v}{\partial z}\Big|_{z=0}\right\}\sin\lambda\alpha\, d\alpha\\ &= \int_0^\infty (\tau_x + i\tau_y)\sin\lambda\alpha\, d\alpha\\ &= \frac{1-\cos(\lambda L/D_h)}{\lambda}(\tau_x + i\tau_y)\end{aligned} \tag{14}$$

if τ_x and τ_y are independent of x. Substituting (10), (11), and (12) into (9) and writing

$$u_1 + i v_1 = W, \tag{15}$$

the two equations (9) can be combined into

$$D_v \frac{d^2 W}{dz^2} - (\lambda^2 + 2\pi^2 i) W - \frac{\pi^2 g}{\omega \sin\varphi} \gamma(\lambda) = 0 \tag{16}$$

and the conditions to be satisfied along the boundaries now become

$$-A_v \frac{dW}{dz}\bigg|_{z=0} = \frac{1-\cos(\lambda L/D_\lambda)}{\lambda} (\tau_x + i\tau_y) \tag{17}$$

and

$$W\big|_{z=h} = 0 \tag{18}$$

The solution of equation (16) subject to conditions (17) and (18) is

$$W = -\frac{\pi^2 g \gamma(\lambda)}{(\lambda^2 + 2\pi^2 i)\omega \sin\varphi} \cdot \left\{ 1 - \frac{\cosh(\sqrt{\lambda^2 + 2\pi^2 i} \cdot z/D_v)}{\cosh(\sqrt{\lambda^2 + 2\pi^2 i} \cdot h/D_v)} \right\}$$

$$+ \frac{\tau_x + i\tau_y}{\sqrt{\lambda^2 + 2\pi^2 i}} \cdot \frac{D_v}{A_v} \frac{\sinh(\sqrt{\lambda^2 + 2\pi^2 i} \cdot \frac{h-z}{D_v})}{\cosh(\sqrt{\lambda^2 + 2\pi^2 i} \cdot h/D_v)} \frac{1-\cos(\lambda L/D_\lambda)}{\lambda} \tag{19}$$

If we separate the real part P of $\sqrt{\lambda^2+2\pi^2 i}$ from the imaginary part Q, we have

$$P=\sqrt{\frac{\sqrt{\lambda^4+4\pi^4}+\lambda^2}{2}}\ ;\ Q=\sqrt{\frac{\sqrt{\lambda^4+4\pi^4}-\lambda^2}{2}} \tag{20}$$

Thus the real part of $\sqrt{\lambda^2+2\pi^2 i}$ is always greater than π.

III. <u>Relation between the Wind Stress and the Slope of the Water Surface.</u>

Now it may be shown that we can establish a definite relation between the wind stress and the slope of the water surface. In a steady state we have no vertical motion of water on the surface of the sea. So we have

$$z=0:\ w=0$$

since the vertical velocity always vanishes on the bottom. Integrating the equation of continuity (2) with respect to z from the surface down to the bottom, we have

$$\frac{\partial}{\partial x}\int_0^h u\,dz=-w\Big|_{z=0}^{z=h}=0.$$

This means that the integral $\int_0^h u\,dz$ is independent of x, or therefore a constant. But as this integral must vanish along the coast or for $x=0$, we must have

$$\int_0^h u\,dz=0 \tag{21}$$

always. Integrating (19) with respect to z from 0 to h, and equating the real part of the resulting equation to zero, we have the following expression for $\gamma(\lambda)$

$$\gamma(\lambda) = \frac{\tau_x}{\rho g h}\,\frac{1-\cos(\lambda L/D_h)}{\lambda}\cdot\frac{h}{D_v}\times$$

$$\frac{\dfrac{P^2-Q^2}{(P^2+Q^2)^2}\left\{1-\dfrac{\cosh(P\,h/D_v)\cos(Q\cdot h/D_v)}{\sinh^2(P\,h/D_v)+\cos^2(Q\,h/D_v)}\right\}+\dfrac{2PQ}{(P^2+Q^2)^2}\,\dfrac{\sinh(P\cdot h/D_v)\sin(Q\cdot h/D_v)}{\sinh^2(P\cdot h/D_v)+\cos^2(Q\,h/D_v)}}{\dfrac{P^2-Q^2}{(P^2+Q^2)^2}\,\dfrac{h}{D_v}-\dfrac{(P^3-3PQ^2)\sinh(P\cdot h/D_v)\cosh(Q\cdot h/D_v)+(3P^2Q-Q^3)\sinh(Q\cdot h/D_v)\cos(Q\cdot h/D_v)}{(P^2+Q^2)^3\{\sinh^2(P\cdot h/D_v)-\cos^2(Q\cdot h/D_v)\}}}$$

$$+\frac{\tau_x}{\rho g h}\,\frac{1-\cos(\lambda L/D_h)}{\lambda}\cdot\frac{h}{D_v}\times$$

$$\frac{-\dfrac{P^2-Q^2}{(P^2+Q^2)^2}\,\dfrac{\sinh(P\cdot h/D_v)\sin(Q\cdot h/D_v)}{\sinh^2(P\cdot h/D_v)+\cos^2(Q\,h/D_v)}+\dfrac{2PQ}{(P^2+Q^2)^2}\left\{1-\dfrac{\cosh(P\cdot h/D_v)\cos(Q\,h/D_v)}{\sinh^2(P\cdot h/D_v)+\cos(Q\cdot h/D_v)}\right\}}{\dfrac{P^2-Q^2}{(P^2+Q^2)^2}\,\dfrac{h}{D_v}-\dfrac{(P^3-3PQ^2)\sinh(P\,h/D_v)\cosh(Q\,h/D_v)+(3PQ-Q^3)\sin(Q\,h/D_v)\cos(Q\cdot h/D_v)}{(P^2+Q^2)^3\{\sinh^2(P\cdot h/D_v)+\cos^2(Q\,h/D_v)\}}}$$

(22)

If we substitute this expression in (12) or

$$\frac{d\zeta}{dx} = \frac{2}{\pi}\int_0^\infty \gamma(\lambda)\sin\lambda\frac{x}{D_h}\,d\lambda, \tag{23}$$

we have the surface slope as induced by the wind stress whose components are τ_x and τ_y respectively.

Once the expression for $\gamma(\lambda)$ is determined, we can obtain u_1 and v_1 by substituting $\gamma(\lambda)$ in (19). Further substitutions of u_1 and v_1 in expressions (10) and (11) will give the horizontal components of velocity. The vertical velocity can be obtained from

$$w = -\frac{\partial}{\partial x}\int_0^z u\,dz \tag{24}$$

an equation derived by integrating the equation of continuity (2) from the surface down to a depth γ .

The preceding analysis covers the principal part of the theory of upwelling discussed by the author recently as a special case when the depth of the sea is very large. In that case we had $\gamma(\lambda) \rightarrow 0$ so only the second term in the right-hand member of (19) was considered. A complete numerical computation involving three components of velocity and the surface slope will be achieved only after a tedious work of very long period. We shall give in this report only the computations as to how the slope of the sea surface varies as we go away from the coast. The author wants to express his desire to extend the computations to all three components of the motion of water in the future because this promises a great number of practical applications. The comparison of computed motion to that actually observed will enable us to estimate the approximate intensity of both vertical and horizontal turbulence in the sea, thus making it possible to predict the wind currents in the sea more accurately.

IV. Computation of Sea Surface Slope and Horizontal Variation of Sea Level.

It is a question of practical calculations to carry the analysis to numerical results. A rather elaborate computation has been carried out by the author during the summer of 1953 when he stayed in the Department of Oceanography, Agricultural and Mechanical College of Texas.

The greater part of the work consisted of numerical evaluation of the function $\gamma(\lambda)$ as given by (22). Because the components of wind stress are given in advance we have only to evaluate the two functions $K\left(\frac{h}{D_v}, \lambda\right)$ and $L\left(\frac{h}{D_v}, \lambda\right)$ given by (26) and (27). They have been computed for

$$\frac{h}{D_v} = 1/16, 1/8, 1/4, 1/2, 1, 2 \text{ and } 4$$

Table I

Offshore Slope of the Sea Surface $\frac{\partial \zeta}{\partial x}$ induced By a Wind Perpendicular to the Coast (τ_x), computed at Different Distances L/D_E from the Coast (units: $\tau_x/\rho g h$, $L =$ [illegible] D_E)

x/D_E \ L/D_E =	1/16	1/8	1/4	1/2	1	Offshore Wind Stress τ_x
0	0	0	0	0	0	
0.1	1.5071	1.4883	1.4012	+1.3274	+1.6308	τ_x
0.2	1.5226	1.4925	1.4720	+1.3203	+0.9259	τ_x
0.3	1.5239	1.4984	1.4791	+1.3076	+0.6871	τ_x
0.4	1.4999	1.4970	1.4467	+1.3270	+1.2606	τ_x
0.5	0.7503	+0.7413	+0.7425	+0.6651	+0.3380	1/2 τ_x
0.6	-0.0046	+0.0091	+0.0362	-0.0511	-0.8992	0
0.7	-0.0069	+0.0088	+0.0064	-0.0324	-0.4360	0
0.8	-0.0096	-0.0065	-0.0001	+0.0027	+0.0015	0
0.9	+0.0030	+0.0002	-0.0001	-0.0163	-0.3723	0
1.0	-0.0040	+0.0054	+0.0005	-0.0318	-0.2729	0
1.1	-0.0007	-0.0044	+0.0007	+0.0354	+0.2424	0
1.2	-0.0016	-0.0049	-0.0012	+0.0088	+0.0812	0
1.3	+0.0007	+0.0046	-0.0004	-0.0217	-0.1910	0
1.4	+0.0000	-0.0042	+0.0000	-0.0077	+0.0483	0
1.5	+0.0076	+0.0003	+0.0022	+0.0157	+0.1036	0

Table II

Offshore Slope of the Sea Surface, $\frac{d\zeta}{dx}$ induced by a Wind Parallel to the Coast (τ_y), Computed at Different Distances x/D_h from the coast.
(unit: $\tau_y/\rho g h$, $L = \frac{1}{2} D_h$)

x/D_h \ $\frac{h}{D_v}$ =	1/16	1/8	1/4	1/2	1	Longshore Wind Stress
0	0	0	0	0	0	τ_y
0.1	0.0441	0.1347	1.2577	-0.2173	-1.8402	τ_y
0.2	0.0477	0.1745	1.5085	+1.1535	+1.9789	τ_y
0.3	0.0481	0.1813	1.5815	+1.6083	+3.2510	τ_y
0.4	0.0460	0.1622	1.4518	+0.6742	+0.3867	τ_y
0.5	0.0241	0.0958	0.8431	+0.9606	1.8433	1/2 τ_y
0.6	0.0021	0.0292	0.2663	1.6698	4.7393	0
0.7	0.0004	0.0089	0.1163	0.7947	2.2709	0
0.8	-0.0001	0.0011	0.0179	0.0527	0.0622	0
0.9	0.0001	0.0013	0.0512	0.5501	1.7700	0
1.0	-0.0001	0.0004	0.0335	0.3901	1.2799	0
1.1	0.0000	-0.0007	-0.0326	-0.3542	-1.1416	0
1.2	-0.0002	-0.0012	-0.0104	-0.1181	-0.3825	0
1.3	0.0000	+0.0005	+0.0248	+0.2778	+0.9025	0
1.4	0.0000	-0.0002	-0.0067	-0.0714	-0.2291	0
1.5	0.0002	-0.0008	-0.0130	-0.1493	-0.4765	0

and for a number of necessary values of δ.

V. Discussion of the Earth's Rotation as a Factor Influencing the Slope.

Expression (23) can be written

$$\frac{d\zeta}{dx} = \frac{\tau_x}{\rho g h}\int_0^\infty K\left(\frac{h}{D_v},\lambda\right)\frac{1-\cos(\lambda L/D_h)}{\lambda}\sin\lambda\frac{x}{D_h}\,d\lambda$$
$$+\frac{\tau_y}{\rho g h}\int_0^\infty L\left(\frac{h}{D_v},\lambda\right)\frac{1-\cos(\lambda L/D_h)}{\lambda}\sin\lambda\frac{x}{D_h}\,d\lambda \qquad (25)$$

where

$$K\left(\frac{h}{D_v},\lambda\right) = \frac{2}{\pi}\,\frac{(1-F)f+G\cdot g}{\frac{h}{D_v}\cdot f - M\cdot m - N\cdot n}\cdot\frac{h}{D_v}, \qquad (26)$$

$$L\left(\frac{h}{D_v},\lambda\right) = \frac{2}{\pi}\,\frac{(1-F)\cdot g - G\cdot f}{\frac{h}{D_v} f - M\cdot m - N\cdot n}\cdot\frac{h}{D_v}, \qquad (27)$$

and

$$F\left(\frac{h}{D_v},\lambda\right) = \frac{\cosh(P\cdot h/D_v)\cos(Q\cdot h/D_v)}{\sinh^2(P\cdot h/D_v)+\cos^2(Q\cdot h/D_v)}, \qquad (28)$$

$$G\left(\frac{h}{D_v},\lambda\right) = \frac{\sinh(P\cdot h/D_v)\sin(Q\cdot h/D_v)}{\sinh^2(P\cdot h/D_v)+\cos^2(Q\cdot h/D_v)} \qquad (29)$$

$$M\left(\frac{h}{D_v},\lambda\right) = \frac{\sinh(P\cdot h/D_v)\cosh(P\cdot h/D_v)}{\sinh^2(P\, h/D_v) + \cos^2(Q\cdot h/D_v)}\,, \tag{30}$$

$$N\left(\frac{h}{D_v},\lambda\right) = \frac{\sin(Q\cdot h/D_v)\cos(Q\cdot h/D_v)}{\sinh^2(P\cdot h/D_v) + \cos^2(Q\cdot h/D_v)}\,, \tag{31}$$

and

$$f(\lambda) = \frac{P^2 - Q^2}{(P^2 + Q^2)^2}\,, \tag{32}$$

$$g(\lambda) = \frac{2PQ}{(P^2+Q^2)^2} \tag{33}$$

$$m(\lambda) = \frac{P^3 - 3PQ^2}{(P^2+Q^2)^3} \tag{34}$$

$$n(\lambda) = \frac{3P^2Q - Q^3}{(P^2+Q^2)^3} \tag{35}$$

and

$$P = \sqrt{\frac{\sqrt{\lambda^4 + 4\pi^4} + \lambda^2}{2}}$$

$$Q = \sqrt{\frac{\sqrt{\lambda^4 + 4\pi^4} - \lambda^2}{2}}$$

being real and imaginary parts of $\sqrt{\lambda^2 + 2\pi i^{2}}$ respectively.

Thus $K\left(\frac{h}{D_v}, \lambda\right)$ and $L\left(\frac{h}{D_v}, \lambda\right)$ depend upon $\frac{h}{D_v}$ and λ only, while τ_x, τ_y, ρ, g and h are of course given quantities.

Next we have

$$\frac{1-\cos(\lambda L/D_h)}{\lambda} \sin \lambda x/D_h$$

$$= \frac{\sin(\lambda \cdot x/D_h)}{\lambda} - \frac{1}{2} \frac{\sin\{\lambda(x-L)/D_h\}}{\lambda} - \frac{1}{2} \frac{\sin\{\lambda(x+L)/D_h\}}{\lambda}$$

$$= \sum_{i=1,2,3} q_i \frac{\sin p_i \lambda}{\lambda} \tag{36}$$

where

$$p_1 = x/D_h, \quad p_2 = (x-L)/D_h, \quad p_3 = (x+L)/D_h,$$

$$q_1 = 1, \qquad q_2 = q_3 = -\frac{1}{2} \tag{37}$$

Figures 2 and 3 give the graphs of the functions $K\left(\frac{h}{D_v}, \lambda\right)$ and $L\left(\frac{h}{D_v}, \lambda\right)$ respectively. These two functions have been estimated for values of

$$\frac{h}{D_v} = 1/16,\ 1/8,\ 1/4,\ 1/2,\ 1,\ 2, \text{ and } 4$$

and some values of λ necessary for furthering the computations. Only the curves for $h/D_v = 1/16,\ 1/8,\ 1/4,\ 1/2,$ and 1 are given in these figures.

From these diagrams we recognize that the value of the function

Table III

Sea Level Difference between the Coast and a Point Distant x from the Coast, induced by an Offshore Wind Stress τ_x. (Unit: $\tau_x D_T/\rho g h$)

x/D_T \ $\tilde{x}/D_T$ =	1/16	1/8	1/4	1/2	1	Offshore Wind Stress
0	0	0	0	0	0	τ_x
0.1	0.075	0.074	0.071	0.066	0.082	τ_x
0.2	0.227	0.223	0.215	0.199	0.209	τ_x
0.3	0.379	0.373	0.362	0.330	0.290	τ_x
0.4	0.530	0.523	0.509	0.462	0.387	τ_x
0.5	0.643	0.635	0.618	0.512	0.467	1/2 τ_x
0.6	0.680	0.672	0.657	0.592	0.439	0
0.7	0.680	0.673	0.659	0.588	0.373	0
0.8	0.679	0.673	0.659	0.587	0.351	0
0.9	0.678	0.673	0.659	0.586	0.332	0
1.0	0.678	0.673	0.659	0.583	0.300	0
1.1	0.678	0.673	0.659	0.584	0.299	0
1.2	0.678	0.673	0.659	0.586	0.315	0
1.3	0.678	0.673	0.659	0.582	0.309	0
1.4	0.678	0.673	0.659	0.584	0.302	0
1.5	0.678	0.673	0.659	0.584	0.310	0

Table IV

Sea Level Difference between the Coast and a Point Distant x from the Coast, induced by a Longshore Wind Stress τ_y. (Unit: $\tau_y D_h/\rho g h$)

x/D_h \ $\frac{l}{D_v}$ =	1/16	1/8	1/4	1/2	1	Longshore Wind Stress
0	0	0	0	0	0	τ_y
0.1	0.002	0.007	0.063	-0.011	-0.092	τ_y
0.2	0.007	0.022	0.201	+0.036	-0.085	τ_y
0.3	0.012	0.040	0.356	+0.174	+0.176	τ_y
0.4	0.016	0.057	0.507	+0.288	+0.358	τ_y
						τ_y
0.5	0.020	0.070	0.622	+0.370	+0.470	1/2 τ_y
0.6	0.021	0.076	0.678	+0.501	+0.799	0
0.7	0.022	0.078	0.697	+0.625	+1.150	0
0.8	0.022	0.079	0.703	+0.667	1.266	0
0.9	0.022	0.079	0.707	+0.697	1.358	0
1.0	0.022	0.079	0.711	0.744	1.510	0
1.1	0.022	0.079	0.711	0.746	1.517	0
1.2	0.022	0.079	0.709	0.722	1.441	0
1.3	0.022	0.079	0.710	0.730	1.462	0
1.4	0.022	0.079	0.711	0.741	1.496	0
1.5	0.022	0.079	0.710	0.730	1.461	0

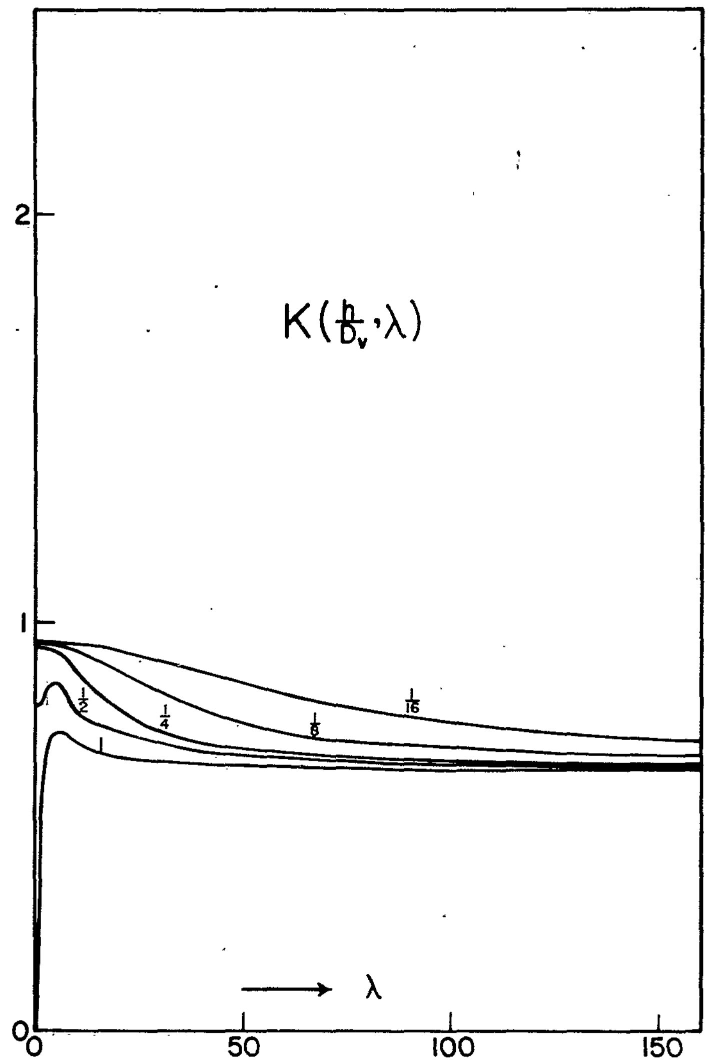

FIG. 2 DEPENDANCE OF THE FUNCTION $K(\frac{h}{D_v},\lambda)$ UPON THE RATIO $\frac{h}{D_v}$.

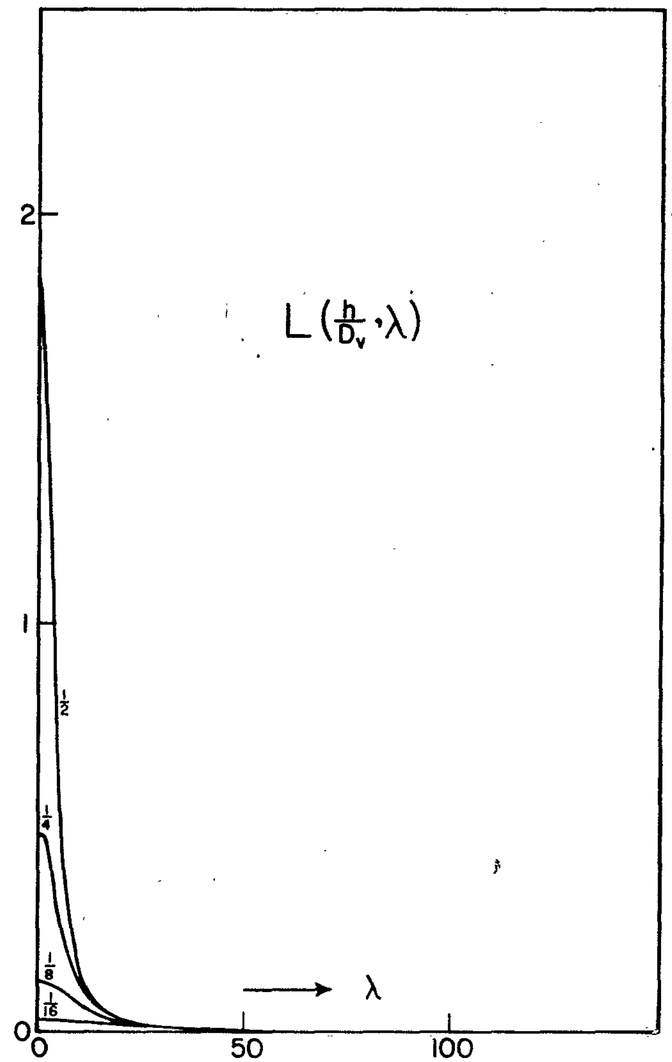

FIG. 3 DEPENDANCE OF THE FUNCTION $L(\frac{h}{D_v},\lambda)$ UPON THE RATIO $\frac{h}{D_v}$.

related to τ_x or the offshore component of wind stress does not show any marked variation for either $\frac{h}{D_v}$ or λ except that its value suddenly falls to zero at $\lambda = 0$ for larger values of h/D_v, while the function $L(\frac{h}{D_v}, \lambda)$ related to τ_y or the longshore component of wind stress has a very large variation. For a smaller value of h/D_v this function varies slightly and smoothly. When h/D_v increases, however, its value at $\lambda = 0$ increases very rapidly. This function thus has always a peak at $\lambda = 0$ and the height of this peak increases proportionally to the square of h/D_v when $\frac{h}{D_v}$ is small and is directly proportional to h/D_v when it is large. At any rate this shows a rapid increase of the function $L(\frac{h}{D_v}, \lambda)$ around $\lambda = 0$ when h/D_v increases from a small value to a larger value.

Now since the function of the type

$$\sin p_i \lambda / \lambda$$

always has a largest value $(= p_i)$ at $\lambda = 0$ and falls rapidly as increases, it can be anticipated that the contributions of the functions $K(h/D_v, \lambda)$ and $L(h/D_v, \lambda)$ to the integrals

$$\int_0^\infty K(h/D_v, \lambda) \frac{\sin p_i \lambda}{\lambda} d\lambda$$

and

$$\int_0^\infty L(h/D_v, \lambda) \frac{\sin p_i \lambda}{\lambda} d\lambda$$

will be largest at $\lambda = 0$. This fact clearly shows that the value of

the integral

$$\int_0^\infty B\left(\frac{h}{D_v}, \lambda\right) \frac{1-\cos(\lambda L/D_h)}{\lambda} \sin \lambda \frac{x}{D_h} \, d\lambda$$

does not vary much with h/D_v, while the integral

$$\int_0^\infty L\left(\frac{h}{D_v}, \lambda\right) \frac{1-\cos(\lambda L/D_h)}{\lambda} \sin \lambda \frac{x}{D_h} \, d\lambda$$

increases greatly as h/D_v increases. In other words, the influence of the earth's rotation is more conspicuous in producing the slope of water surface when it is induced by a wind parallel to the coast than by a wind perpendicular to the coast.

VI. Computation of the Sea Surface Slope.

Now expression (25) becomes

$$\frac{d\zeta}{dx} = \frac{\tau_x}{\rho g h} S_x\left(\frac{h}{D_v}, x\right) + \frac{\tau_y}{\rho g h} S'_y\left(\frac{h}{D_v}, x\right) \tag{38}$$

where

$$S_x\left(\frac{h}{D_v}, x\right) = \int_0^\infty B\left(\frac{h}{D_v}, \lambda\right) \frac{1-\cos(\lambda L/D_h)}{\lambda} \sin \lambda \frac{x}{D_h} \, d\lambda \tag{39}$$

and

$$S'_y\left(\frac{h}{D_v}, x\right) = \int_0^\infty L\left(\frac{h}{D_v}, \lambda\right) \frac{1-\cos(\lambda L/D_h)}{\lambda} \sin \lambda \frac{x}{D_h} \, d\lambda \tag{40}$$

It is quite easy to compute these integrals if the functions

$$X\left(\frac{h}{D_v}, x\right) = \int_0^\infty K\left(\frac{h}{D_v}, \lambda\right) \sin \lambda \frac{x}{D_h} \, d\lambda \tag{41}$$

and

$$Y\left(\frac{h}{D_v}, x\right) = \int_0^\infty L\left(\frac{h}{D_v}, \lambda\right) \sin \lambda \frac{x}{D_h} \, d\lambda \tag{42}$$

are computed and compiled. Tables V and VI give parts of such compilations. For example, when we want to compute the integral (39) for $x/D_h = 0.3$ assuming $L/D_h = 0.5$, we have simply by (36) and (37) to make a sum

$$X\left(\frac{h}{D_v}, 0.3\right) = \frac{1}{2} X\left(\frac{h}{D_v}, 0.3-0.5\right) - \frac{1}{2} X\left(\frac{h}{D_v}, 0.3+0.5\right)$$

$$= \frac{2X(0.3) + X(0.2) - X(0.8)}{2}$$

for a given value of h/D_v, because $X(h/D_v, x)$ is an even function of x. The same applies for the function $Y(h/D_v, x)$ represented by the sums of integrals of the form (42). Tables V and VI will enable us to make computations for other values of the ratio L/D_h.

By this process, we can compute very easily the slope of water surface induced by both offshore and longshore wind stress components.

The following computation was made when the width L of the wind zone is half as large as the frictional distance D_h. So we have $L/D_h = 0.5$

Table V

Function $\chi(\frac{h}{D_v}, x)$ for Computing the Sea Surface Slope for Uniform Wind Stress

x/D_v \ h/D_v =	1/16	1/8	1/4	1/2	1
0	0	0	0	0	0
0.1	1.5104	1.4856	1.4099	1.2737	1.1531
0.2	1.5147	1.4948	1.4705	1.2912	0.6486
0.3	1.5156	1.4972	1.4853	1.2990	0.4594
0.4	1.4939	1.5043	1.4833	1.3276	0.6716
0.5	1.5002	1.4937	1.4838	1.2848	0.2828
0.6	1.5004	1.4989	1.4826	1.2200	-0.2838
0.7	1.4998	1.5039	1.4824	1.2408	-0.0953
0.8	1.4980	1.4926	1.4830	1.2739	0.1933
0.9	1.5000	1.5001	1.4830	1.2748	-0.0248
1.0	1.4997	1.5025	1.4825	1.2394	-0.1104
1.1	1.4995	1.4940	1.4830	1.2687	0.0775
1.2	1.4986	1.4955	1.4814	1.2551	0.0328
1.3	1.4996	1.5009	1.4826	1.2434	-0.0758
1.4	1.5001	1.4955	1.4829	1.2547	+0.0234
1.5	1.5074	1.5006	1.4848	1.2577	+0.0422
1.6	1.8999	1.4980	1.4819	1.2464	-0.0460
1.7	1.5005	1.4969	1.4829	1.2518	-0.0014
1.8	1.4998	1.5000	1.4830	1.2563	+0.0372
1.9	1.5001	1.4993	1.4829	1.2500	-0.0249
2.0	1.4997	1.4980	1.4828	1.2505	-0.0125

Table VI

Function $\Upsilon(\frac{h}{D_v}, x)$ for Computing the Sea Surface Slope for Uniform Wind Stress

x/D_v \ $h/D_v =$	1/16	1/8	1/4	1/2	1
0	0	0	0	0	0
0.1	0.0442	0.1363	1.3230	0.4856	0.4272
0.2	0.0475	0.1762	1.5620	1.6098	3.3560
0.3	0.0484	0.1893	1.6717	2.1035	4.5417
0.4	0.0481	0.1904	1.6760	1.8909	3.6600
0.5	0.0482	0.1923	1.7339	2.4812	5.5289
0.6	0.0482	0.1936	1.8064	3.2966	8.1949
0.7	0.0481	0.1928	1.7787	3.0161	7.2959
0.8	0.0482	0.1922	1.7425	2.6002	5.9374
0.9	0.0482	0.1928	1.7714	2.9189	6.9699
1.0	0.0482	0.1930	1.7817	3.0411	7.3713
1.1	0.0482	0.1925	1.7573	2.7681	6.4840
1.2	0.0482	0.1916	1.7628	2.8330	6.6941
1.3	0.0482	0.1929	1.7774	2.9915	7.2088
1.4	0.0482	0.1926	1.7644	2.8469	6.7397
1.5	0.0482	0.1928	1.7624	2.8209	6.6538
1.6	0.0482	0.1928	1.7735	2.9479	7.0563
1.7	0.0482	0.1927	1.7677	2.8861	6.8573
1.8	0.0482	0.1926	1.7628	2.8273	6.6751
1.9	0.0482	0.1928	1.7709	2.9177	6.9676
2.0	0.0482	0.1942	1.7691	2.8993	6.9094

and the slope of the water surface was computed at several distances from the coast. Both the surface slopes induced by the offshore and longshore wind stress components are given in Tables I and II.

From these results it can be concluded that the slope of the water surface is chiefly found in the wind zone and it is mostly very small outside the latter. However, the manner of increase of the slope of the water surface with increasing ratio h/D_v is much different between the offshore and longshore winds. In case of the offshore wind τ_x the slope induced by it does not vary much with the ratio h/D_v. Their values within the wind zone lie mostly between

$$1.3 \cdot \frac{\tau_x}{\rho g h} \sim 1.3 \cdot \frac{\tau_x}{\rho g h}$$

On the contrary, the slope caused by a longshore wind stress is very small when h/D_v is small and increases very rapidly as this ratio increases. Thus the slope in the wind zone is a little less than 0.05 for $h/D_v = 1/16$. But it increases to almost 25 times when the ratio h/D_v increases to 1/4. At $h/D_v = 1$, the slope varies rather irregularly. This may be because of the incompleteness of the numerical integration and it would be dangerous to believe this result to be very accurate. At any rate the slope increases very rapidly with the ratio h/D_v provided there is a stationary stage around $h/D_v = 0.5$.

From this result it can be concluded that the slope of the sea surface induced by wind stresses is proportional to the wind stress τ_x and τ_y, inversely to the depth h of the sea provided the ratio h/D_v is given. They are nearly independent of the magnitude D_h or the horizontal turbulence.

If we take $\tau_x = 1$, $h = 50$ meters, then we shall have

$$\frac{d\zeta}{dx} = 2.6\times10^{-7} \sim 3.0\times10^{-7}$$

This is a slope about 3 cm per 100 km, or about 2 inches per 100 nautical miles. The stress $\tau_x = 1$ corresponds to a wind of speed about 6 or 7 m/sec. When h = 100 m the slope is half as large.

The fact that the slope is very small when h/D_v is small, means that the influence of the earth's rotation is largely pressed down by the bottom friction. As the depth of the water approaches D_v gradually, the earth's rotation becomes a more and more important factor.

Although these results are all purely theoretical ones, there is no reason why they are of no practical application. Comparison with great numbers of observations will give some idea about the magnitudes of both horizontal and vertical mixing coefficients.

VII. Change of Sea Level in an Offshore Direction.

Determination of the slope of the sea surface enables us to know how the surface of the sea rises or falls as we are removed from the coast. Because the water surface is assumed to neither rise nor fall in a direction parallel to the coast, we have only to check the change of sea level in an offshore direction.

An approximate formula to compute a curve $y = F(x)$ from the values of $\frac{dy}{dx}$ ~~y = F(x)~~ at two points separated by Δx is

$$y_n = y_{n-1} + \frac{1}{2}\left\{\left(\frac{dy}{dx}\right)_{n-1} + \left(\frac{dy}{dx}\right)_n\right\}\cdot\Delta x$$

where $\left(\frac{dy}{dx}\right)_{n-1}$ and $\left(\frac{dy}{dx}\right)_n$ are the values of y at x_{n-1} and x_n

separated by Δx. Assuming we have a water height ζ_0 on the coast, we have for the change of level produced by an offshore stress

$$\zeta_1 = \zeta_0 + \frac{\Delta x}{2} \cdot \{ \varrho S_x(0) + \varrho S_x(\Delta x) \},$$

$$\zeta_2 = \zeta_1 + \frac{\Delta x}{2} \{ \varrho S_x(\Delta x) + \varrho S_x(2\Delta x) \},$$

$$\cdots\cdots$$

and so on. The same applies to the slope induced by longshore stress τ_y. By this process it will be possible for us to derive the sea surface profiles produced by both offshore and longshore wind stresses. Actual sea level consists of the sum of these two. Tables V and VI give the results for both of these stress components respectively. These are also illustrated by Figures 4 and 5.

Looking at Tables III and IV and the two diagrams (Figures 4 and 5) we at once notice that there is practically no change in sea level outside the wind zone within a width L_1 from the coast.

For a longshore wind blowing in such a manner that for an observer looking in the direction of the wind with the sea on his right hand side, the sea level rises nearly linearly as we are removed away from the coast until we arrive at the end of the wind zone. This tendency is common to the cases $\underline{h/D_v}$ = 1/16, 1/8, 1/4 but some irregularities occur when $\underline{h/D_v}$ = 1/2. It will be hard to know if those irregularities really exist or if they are actually due to some incompleteness of the procedure of numerical integration. Perhaps the latter explanation holds better. In any event, the general tendency is that the sea surface outside the wind zone suffers no appreciable level change. Now since the sea is supposed to extend infinitely, the change of the sea level in a finite area will not affect the level in an infinitely

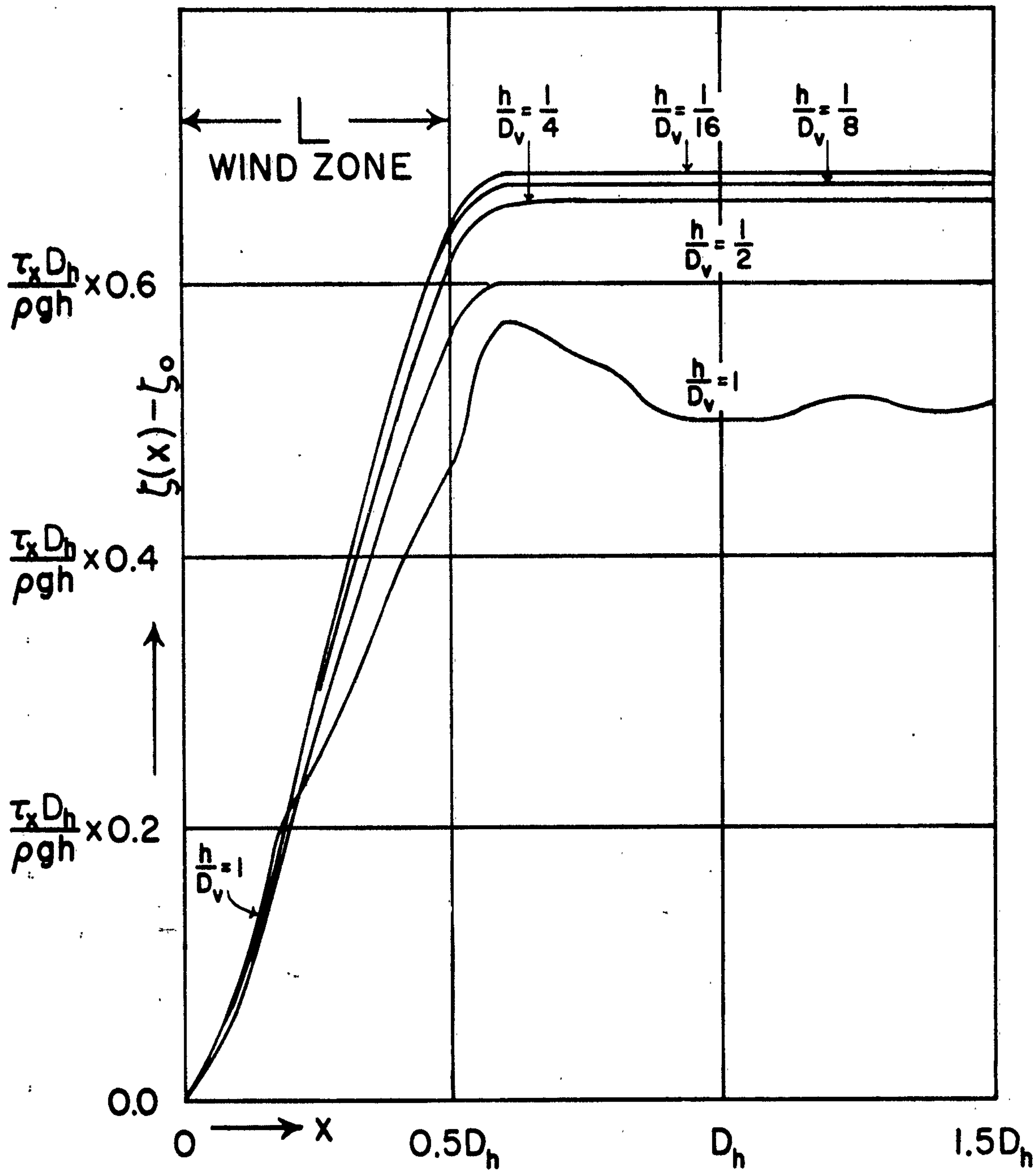

FIG. 4 SEA LEVEL DIFFERENCE BETWEEN THE COAST AND A POINT A DISTANCE x FROM THE COAST PRODUCED BY THE OFFSHORE COMPONENT τ_x OF THE WIND STRESS COMPUTED FOR SEVERAL DIFFERENT VALUES OF $\frac{h}{D_v}$.

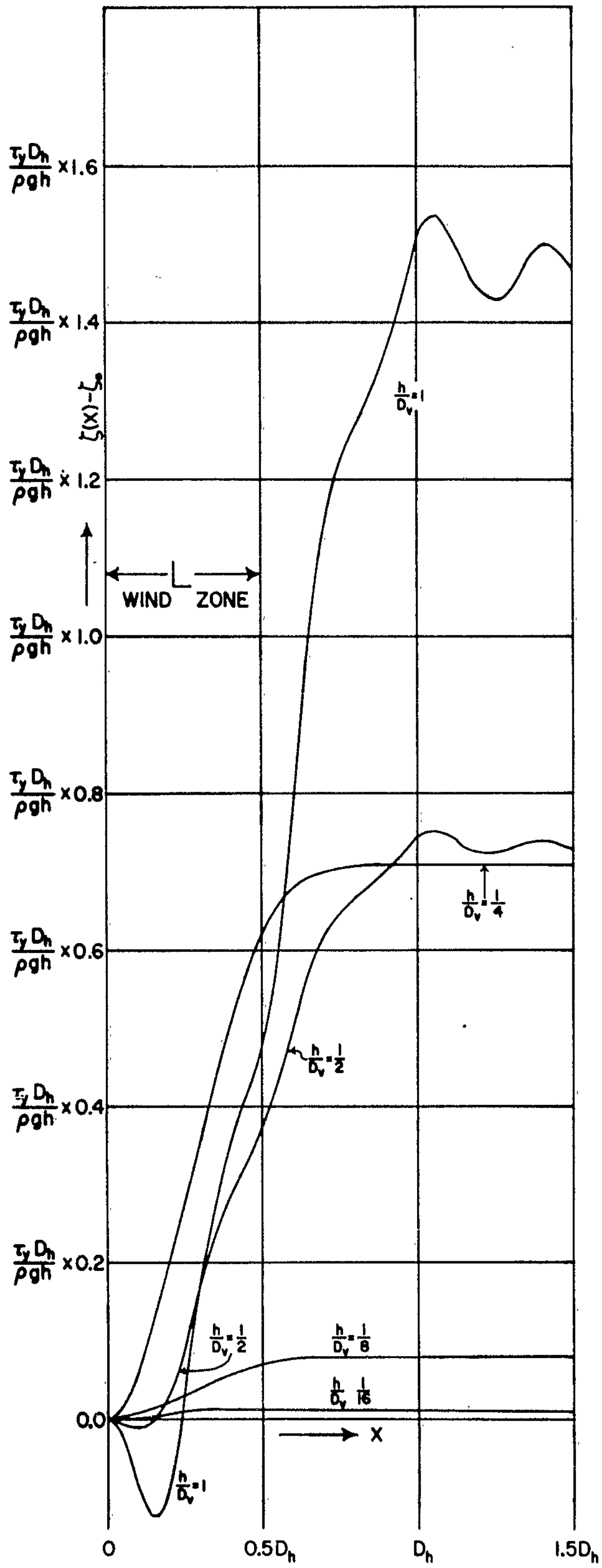

FIG. 5 SEA LEVEL DIFFERENCE BETWEEN THE COAST AND A POINT A DISTANCE x FROM THE COAST PRODUCED BY THE LONGSHORE COMPONENT τ_y OF THE WIND STRESS COMPUTED FOR SEVERAL DIFFERENT VALUES OF $\frac{h}{D_v}$.

wide area outside the wind zone. This means that when the wind blows in the above manner, we can expect a depression of sea level beneath the wind-swept area deepening linearly toward the coast. The maximum level fall occurs of course along the beach. If the wind blows in the opposite direction, there will occur an elevation of the sea surface toward the coast. The magnitude of these depressions and elevations of course depends upon the ratio h/D_v and L, the width of the wind zone.

For an offshore wind blowing in such a manner that the observer looking towards the sea has the wind on his back, the same sort of depression takes place, of course, the manner of its dependence upon h/D_v differing from the case of longshore wind. If the wind blows from the sea to land there will occur an elevation beneath the area swept by the wind.

These details are illustrated by the diagrams in Figure 6.

VIII. Relation Between the Wind Direction and the Sea Level Change.

The diagrams in Figure 6 give us an approximate idea of the relationship between the direction of the wind stress and the sea level change in a steady state. The sea level rises approximately linearly as we are removed away from the coast. No slope of the sea surface is seen outside the wind zone. The sea level responds to the offshore and longshore wind in different ways. For example, in the area of California, a north wind lowers the sea level below the wind zone and a south wind raises it. On the other hand an east wind raises the level and a west wind lowers it. Thus is can be concluded that for some direction of wind and for some ratio h/D_v there will occur neither rise nor fall of the sea level however strong the wind may blow. Such directions will be found in the sectors between north and west and south and east.

On the contrary there will be a wind direction which gives a maximum rise or fall of the sea level. This direction must of course depend upon

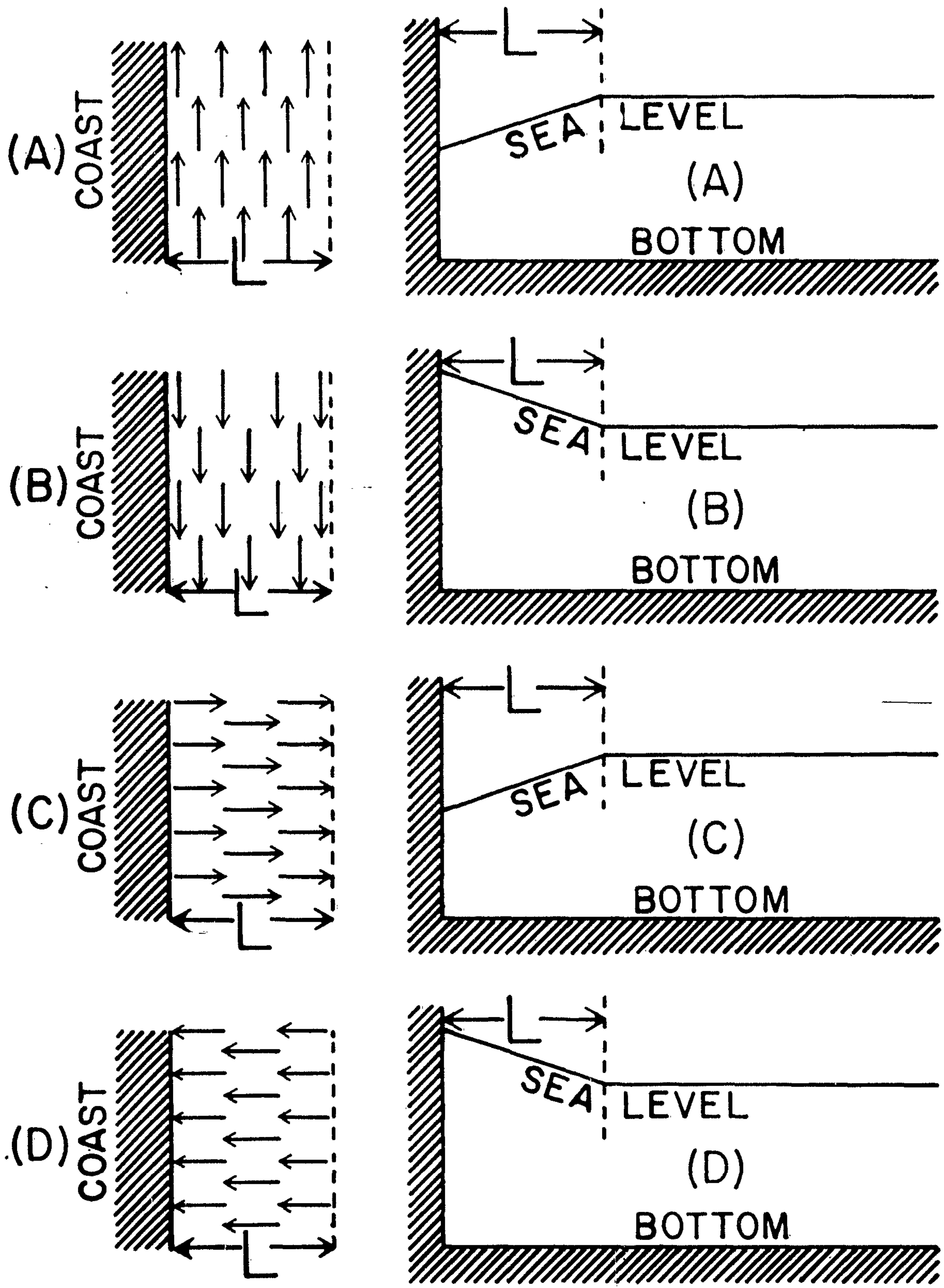

FIG. 6 SCHEMATIC DIAGRAMS SHOWING THE RELATION BETWEEN THE DIRECTION OF WIND AND SEA LEVEL CHANGE.

the ratio h/D_v, that is to say, on the square root of the mixing coefficient, providing the depth to the bottom is given. Off Texas and Louisiana coasts, the wind from east to southeast and from opposite directions will not be effective in raising or lowering the sea level on the continental shelf. On the contrary north or south winds are expected to produce strong falls or rises in the sea level on the shelf.

Summary. The theory of the wind-driven currents in a shallow sea is considered taking into account the effect of horizontal momentum transfer. Other assumptions and conditions are nearly similar to Ekman's work except that we assume an infinite straight vertical barrier for the coast. The complete solution involving the expressions for the three components of velocity and the variation of the surface slope at different distances from the coast appears to take a very long time and require tedious computations. For this reason only the result for the slopes of the sea surface is given in this paper. The following conclusions have been drawn.

(1) Due to the stress of wind there occurs a rise or fall of the sea surface. When the wind blows within a finite zone from the coast, this surface slope occurs only in this zone and no slope is seen outside it.

(2) When the wind is uniform, the sea surface within the wind zone rises or falls linearly toward the coast.

(3) For a certain wind direction and for certain values of the ratio h/D_v, no rise or fall of the sea level will occur. On the contrary, there will be directions of winds for which we have a maximum rise or fall of the sea level. These features will depend upon the direction of wind, depth to the bottom latitude and the vertical mixing coefficient.

(4) Complete numerical solution of this problem for the three dimensional water movement is intended by the author for a future opportunity.

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